Chin. J. Astron. Astrophys. Vol. 0, No. 0, (200x) 000–000

Chinese Journal of Astronomy and Astrophysics

Dark Matter and Dark Energy: a Scenario of Accelerating Universe

Utpal Mukhopadhyay[1] and Saibal Ray[2,3 ⋆]

[1]Satyabharati Vidyapith, Barasat, Kolkata 700 126, North 24 Pgs, W. B., India
[2]Physics Department, Barasat Govt. College, Kolkata 700 124, North 24 Pgs, W. B., India
[3]Inter-University Centre for Astronomy and Astrophysics, PO Box 4, Pune 411 007, India

Received 2005 ; accepted 2005

Abstract Dark matter, the major component of the matter content of the Universe, played a significant role at early stages during structure formation. But at present the Universe is dark energy dominated as well as accelerating. Here, the presence of dark matter has been established by including a time-dependent Λ term in the field equations. This model is compatible with the idea of an accelerating Universe so far as the value of the deceleration parameter is concerned.

Key words: cosmology: theory - cosmological parameters - Dark matter

pacs: 04.20.-q, 04.20.Jb, 98.80.Jk

1 INTRODUCTION

Dark matter and dark energy are two major constituents of the present Universe. As it stands today, the Universe is composed of nearly 30% matter and 70% dark energy (Kirshner 2003). Again, of the total matter content, about 25% are non-luminous or dark while visible component of myriads of galaxies contribute a meager 5% (Shull 2005; Ostriker & Steinhardt 2003). Long ago, using Virial Theorem, Zwicky (1937) for the first time suggested about the existence of dark matter. Afterwards, galactic rotation curve studies also supported Zwicky's idea (Roberts

⋆ E-mail: saibal@iucaa.ernet.in

& Rots 1973; Ostriker, Peebles & Yahil 1974; Einasto, Kaasik & Saar 1974; Rubin, Thonnard & Ford (Jr) 1978). When, as a consequence of Inflationary Theory of Guth (1981), it became clear that the Universe must be flat, cosmologists became convinced that 96% matter content of the Universe should be hidden mass. But, this simplified cosmological picture soon ran into trouble since, in spite of an intense search, evidence in favor of such a huge amount of dark matter was lacking. Consequently, the future of Inflationary Theory was at stake. Theoretical scientists then speculated that matter energy density of the Universe cannot be more than one-third of the total energy density and hence the remaining two-third energy density should be compensated by a cosmological constant (Turner 2003). Finally, observational result for an accelerating Universe (Perlmutter et al 1998; Riess et al 1998) favored the above speculation and the idea of an accelerating agent, termed as dark energy, was accepted.

Various dark energy models have been proposed during the last five years or so (for an overview see (Overduin & Cooperstock 1998) and (Sahni & Starobinsky 2000). One of the favorite candidates among these, obviously, are the models related to dynamic Λ-term. In this paper, taking phenomenological Λ models into consideration, existence of dark matter has been established. Moreover, it has been possible to show that the present models support the idea of an accelerating universe. Here Sec. 2 deals with the field equations with Λ term while values of various cosmological parameters have been calculated in Sec. 3. Finally, merits of the present models have been discussed in Sec. 4.

2 FIELD EQUATIONS AND GENERAL RESULTS

We know that Einstein's field equation (including cosmological parameter Λ) are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right]. \tag{1}$$

For the spherically symmetric FLRW metric the above equation yield respectively Friedmann equation and Raychaudhuri equation given by

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3}, \tag{2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{3}$$

where $\Lambda = \Lambda(t)$ is time-dependent, the velocity of light c is unity in relativistic units, k is the curvature constant and $a(t)$ is the scale factor of the Universe.

From (2) we have

$$\rho = \frac{3}{8\pi G}\left(\frac{k}{a^2} + H^2 - \frac{\Lambda}{3}\right). \tag{4}$$

So, the present energy density is given by

$$\rho_0 = \frac{3}{8\pi G}\left(\frac{k}{a_0^2} + H_0^2 - \frac{\Lambda_0}{3}\right) \tag{5}$$

where the suffix zero indicates the present values of the corresponding cosmological parameters. Again, the deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H}\left(\frac{\ddot{a}}{a}\right). \tag{6}$$

Therefore,

$$\frac{\ddot{a}}{a} = -qH. \tag{7}$$

Using (7) and (4), we get from (3),

$$-qH = -\frac{1}{2}\left(\frac{k}{a^2} + H^2 - \frac{\Lambda}{3} - 4\pi Gp + \frac{\Lambda}{3}\right) \tag{8}$$

which, after simplification, provides the expression for pressure given by

$$p = -\frac{1}{8\pi G}\left[\frac{k}{a^2} + (1-2q)H^2 - \Lambda\right]. \tag{9}$$

Thus, the present value for the pressure is given by

$$p_0 = -\frac{1}{8\pi G}\left[\frac{k}{a_0^2} + (1-2q_0)H_0^2 - \Lambda_0\right]. \tag{10}$$

Let us choose the barotropic equation of state

$$p = \omega\rho \tag{11}$$

where ω is the barotropic index or equation of state parameter such that $\omega = p/\rho$. In general, ω is a function of time, scale factor or redshift but sometimes it is convenient to consider ω as a constant quantity because current observational data has limited power to distinguish between a time varying and constant equation of state (Kujat et al 2002; Bartelmann et al 2005). However, we shall see in Sec. 3.3 that assumption of ω as constant in time will provide interesting physical scenario.

For flat ($k = 0$) Universe, using equations (4) and (9) we get from equation (11)

$$\omega = \frac{3\Omega_\Lambda + 2q - 1}{3(1-\Omega_\Lambda)} \tag{12}$$

where $\Omega_\Lambda = \Lambda/3H^2$ is the vacuum energy density of the Universe. Similarly, for flat Universe, equation (5) and (10) can respectively be written as

$$\rho_0 = \frac{3H_0^2}{8\pi G}(1 - \Omega_{\Lambda_0}) \tag{13}$$

and

$$p_0 = -\frac{H_0^2}{8\pi G}[(1 - 2q_0) - 3\Omega_{\Lambda_0}] \tag{14}$$

where Ω_{Λ_0} is the present value of the vacuum energy density.
From equations (12), (13) and (14) it is clear that the fate of the Universe depends on q, Ω_Λ and H. Also it is clear from equation (13) that for physical reality Ω_{Λ_0} must be less than one. We also know that the case $p_0 > 0$ provides a collapsing Universe. So, equation (14) tells us that for a collapsing Universe, we must have $(1 - 2q_0) > 3\Omega_{\Lambda_0}$.

3 PHYSICAL FEATURES OF THE MODEL

3.1 Calculation of ρ_c, ρ_0 and ρ_G

An important quantity which determines the future of the Universe is the critical density ρ_c. The Universe is open or closed according as the present density ρ_0 of the Universe is less or greater than the critical density. Since at present, the accepted value of the Hubble parameter H_0 is $(72 \pm 8) kms^{-1}Mpc^{-1}$ (Kirshner 2003), we may choose $H_0 = 72 kms^{-1}Mpc^{-1}$ for our calculation. For this value of H_0, we get $\rho_c = 3H_0^2/8\pi G \sim 9 \times 10^{-30} gmcm^{-3}$.
In one of our previous work (Ray & Mukhopadhyay 2004) we have shown that the three kinematical Λ models, viz. $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim (\ddot{a}/a)$ and $\Lambda \sim \rho$ are equivalent and for these models, $\rho_0 = 3 \times 10^{-30} gmcm^{-3}$. Also, the measured galactic mass density ρ_G is given by (Deb 1999), $\rho_G = 3.1 \times 10^{-31} gmcm^{-3}$. Therefore,

$$\frac{\rho_G}{\rho_c} \sim 0.033 \tag{15}$$

Also,

$$\frac{\rho_0}{\rho_c} = \frac{1}{3} \tag{16}$$

From (15) and (16) we, immediately, obtain

$$\rho_G \sim 0.1\rho_0 \tag{17}$$

It is clear from equation (17) that galactic mass density is about 10% of the total mass density of the present Universe. Hence, there must be some hidden mass. Also, equation (16) implies that the present total density of the Universe is one-third of the critical density. This means that galactic (luminous) mass-density is about 3% of the critical density.
Again, equation (5) can be written as

$$\frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{\Lambda_0}{3} - (\rho_c - \rho_0)\right] \tag{18}$$

From equation (16) it is easy to see that $\rho_0 < \rho_c$ and hence $(\rho_c - \rho_0) > 0$. Also, present observational results indicate that, the Universe is flat (k=0). So, for a flat Universe we must have,

$$\Lambda_0 = 3(\rho_c - \rho_0) \tag{19}$$

On the other hand, for a closed Universe, $\Lambda_0 > 3(\rho_c - \rho_0)$ whereas for an open Universe, $\Lambda_0 < 3(\rho_c - \rho_0)$. So, the cosmological parameter is an important factor for determining the geometry of the Universe. Now, one of the predictions of the inflationary theory is a flat Universe and Λ had a large value in the early stages of the Universe. So, one may argue that it is the cosmological parameter which made the Universe flat during inflation.

3.2 Calculation of q_0

For pressureless non-relativistic matter, $p = 0$. Then from equation (10) we have,

$$\frac{k}{a^2} = \Lambda_0 - (1 - 2q_0)H_0^2 \tag{20}$$

Using equation (20), we get from equation (5),

$$\rho_0 = \frac{1}{4\pi G}(\Lambda_0 + 3q_0 H_0^2). \tag{21}$$

Therefore,

$$\frac{\rho_0}{\rho_c} = 2\left(q_0 + \frac{\Lambda_0}{3H_0^2}\right) = 2(q_0 + \Omega_{\Lambda_0}) \tag{22}$$

Using equation (16) and noting that Ω_{Λ_0} is nearly equal to 0.7 we get from equation (22) that q_0 is about -0.53. This value of q_0 is in excellent agreement with the present accepted value of this parameter (Sahni & Starobinsky 2000) and represents an accelerating Universe. Moreover, Deb (1999), without taking into account the cosmological parameter Λ, showed that q_0 must be positive (equation (6) of Deb (1999)), whereas inclusion of Λ has presented us a situation in

which we can suggest that q must be negative. This indicates the inconsistency of the result of Deb (1999) and the present work can be regarded as an improvement over that so far as the present status of the Universe is concerned.

3.3 Calculation of ω

It is mentioned earlier that, as a simplest case it is useful to model dark energy cosmology with a constant equation of state parameter ω (Kujat et al 2002; Bartelmann et al 2005). However, some useful limits on ω was suggested by SNIa data, $-1.67 < \omega < -0.62$ (Knop et al 2003) whereas refined values come from combined SNIa data with CMB anisotropy and galaxy clustering statistics which is $-1.33 < \omega < -0.79$ (Tegmark et al 2004). Therefore, let us calculate the value of ω as governed by the value of q_0 and Ω_{Λ_0} in the present case. Putting $q_0 = -0.53$ and $\Omega_\Lambda = 0.7$, we obtain from equation (12), $\omega = 0.044$. So, if $q_0 = -0.53$, then $\omega > 0$ and hence $p > 0$. Thus, the present accelerating Universe may re-collapse in future if the present value of the deceleration parameter is not greater than -0.53. If $q_0 = -0.55$, then we get $p = 0$ and hence a dust-filled Universe. It should be also noted here that for quintessence, vacuum fluid and phantom energy, the rate of acceleration should be higher. For instance, when $\omega = -0.5$, -1.0 and -2.0 (note that for their simulations Kuhlen et al (2005) consider a range of parameter space: $\omega = -0.5$, -0.75, -1.0, -1.25 and -1.5) then we get respectively, $q_0 = -0.775$, -1.0 and -1.45. On the other hand, for stiff fluid ($\omega = 1.0$), $q_0 = -0.1$. So, more smaller the value of ω, higher is the rate of acceleration. This higher acceleration may produce the so-called Big Rip (Caldwell, Kamionkowsky & Weinberg 2003) or Partial Rip (Štefančić 2004) scenario due to divergence of scale factor. So, in the present paper, through an indirect approach, it has been possible to arrive at the two interesting physical ideas of modern cosmology mentioned above.

4 CONCLUSIONS

Ever since the inception of the idea of dark energy, the cosmological scenario has changed significantly because this exotic type of energy, in the absence of any curvature, has become the dominant energy component of the Universe. In the present work, including the cosmological term Λ in the field equations it has been possible to show the existence of dark matter which shares a large portion of matter content of the Universe. Results of this investigation are compatible with the present idea of an accelerating Universe and clearly demonstrates the

limitations of the work of Deb (1999). Without including the Λ term, Deb arrived at a situation where q_0 cannot be negative (equation (6) of Deb) and hence cannot present us an accelerating Universe, whereas the present work, without any special assumption, gives us a value of q_0 which agrees very well with the accepted value of that parameter. Moreover, the crucial role of the equation of state parameter ω has been clearly demonstrated via equation (12). It has also been possible to show that if the rate of acceleration lie below a limit then the Universe may re-collapse in future.

Acknowledgements One of the authors (SR) would like to express his gratitude to the authority of IUCAA, Pune for providing him the Associateship Programme under which a part of this work was carried out.

REFERENCES

Bartelmann M. et al, 2005, New Astron. Rev., 49, 199

Caldwell R. R., Kamionkowski M. and Weinberg N. N., 2003, Phys. Rev. Lett., 91, 071301

Deb R. N., 1999, Ind. J. Phys. B, 73, 6

Einasto J., Kaasik A. and Saar E., 1974, Nat., 250, 309

Guth A. H., 1981, Phys. Rev. D, 23, 347

Kirshner R. P., 2003, Science, 300, 5627

Knop R. et al, 2003, Ap. J., 598, 102

Kuhlen M. et al, 2005, Mon. Not. R. Astron. Soc., 357, 387

Kujat J. et al, 2002, Ap. J., 572, 1

Ostriker J. P., Peebles P. J. E. and Yahil A., 1974, Ap. J. Lett., 193, L1

Ostriker J. P. and Steinhardt P., 2003, Science, 300, 1909

Overduin J. M. and Cooperstock F. I., 1998, Phys. Rev. D, 58, 043506

Perlmutter S. et al, 1998, Nat., 391, 51

Ray S. and Mukhopadyay U., astro-ph/0407295

Riess A. G. et al, 1998, Astron. J., 116, 1009

Roberts M. S. and Rots A. H., 1973, Astron. Astrophys., 26, 483

Rubin V. C., Thonnard N. and Ford W. A. (Jr), 1978, Ap. J. Lett., 255, L107

Sahni V. and Starobinsky A., 2000, Int. J. Mod. Phys. D, 9, 373

Shull J. M., 2005, Nat., 433, 465

Štefančić H., 2004, Phys. Lett. B, 595, 9

Tegmark M. et al, 2004, Ap. J., 606, 702

M. S. Turner M. S., 2003, Phys. Today, April

Zwicky F., 1937, Ap. J., 86, 217